Promotion and Employee Spinoffs

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We construct a simple dynamic model of promotion and spinoff formation that sheds some light on why the empirical relationships between spinoff formation, tenure and performance vary so widely across different settings. A supervisor must learn over time about the employee's aptitude for work at a more senior level and decide whether to promote him. The employee balances the benefits of waiting for promotion against immediate departure to form a spinoff. By means of a number of approximations to the pair of interrelated optimal stopping problems that our model gives rise to, we are able to characterize the effects of parameter changes on the likelihood and timing of promotion and spinoff formation.

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1. Introduction

How does the likelihood of engaging in entrepreneurship change as an individual acquires experience at existing organizations? The answer is less intuitive than it may, at least at first, seem to be. On the one hand, if employees learn from successful employers about how to compete profitably in their industry (e.g., Agarwal et al. [2004], Franco and Filson [2006]), one might expect the rate at which an individual transitions to entrepreneurship to increase with experience. On the other hand, the accumulation of experience at existing organizations can deter entrepreneurship in either of two ways. First, experience may raise human capital specific to the firm or, more generally, specific to work done in large organizations. If the employee is able to capture part of this increased value in the form of higher wages, the opportunity cost of entrepreneurship rises. Second, tenure may indicate an individual's preference or innate aptitude for continued employment over entrepreneurship, either at established organizations in general (e.g., Sørensen [2007], Elfenbein, Hamilton, and Zenger [2010]), or at his current place of employment in particular (e.g., Jovanovic [1979]).

As a consequence, it is perhaps unsurprising that evidence on the empirical relationship between the accumulation of experience and the rate of entrepreneurship is mixed. For example, a study of academic scientists found that the rate of transition to commercial science increased monotonically but non-linearly with experience, as indicated by publication counts and number of jobs held (Stuart and Ding [2006]). A study of MBA graduates indicated a non-monotonic relationship between organizational tenure and entrepreneurship that changed direction twice over the tenure distribution (Dobrev and Barnett [2005]). Another study, of Danish citizens, found that the rate of transition to entrepreneurship decreased with one's tenure at their current employer (Sørensen [2007]). Last, a study of lawyers found a non-monotonic (first increasing and then decreasing) relationship between tenure and the rate at which lawyers departed their employer to found a new firm (Campbell, Ganco, Franco, and Agarwal [2012]).

Ambiguities about the effect of tenure extend to ambiguities about the effect of job performance on entrepreneurship. While high performance on the job may reflect high ability that is most effectively rewarded by business formation (Rosen [1981]), it also raises the opportunity cost of leaving a firm. On this score, too, the empirical evidence is mixed. For example, Evans and Jovanovic [1989] provide evidence that people leaving a firm on average had performed worse than those who stay; Elfenbein, *et al.* [2010] and Astebro, Chen and Thompson [2010] found that the highest and lowest earners are more likely to enter self-employment; while Groysberg, Nanda and Prats [2009] found that star financial analysts were more likely to leave their employer to engage in entrepreneurship even though they were less likely to leave in order to work at another firm.

In this paper, we construct a simple model intended to shed some light on why the relationships between spinoff formation, tenure and performance vary so widely across different settings. In our model there are two agents, a Supervisor (S) and a Junior (J). S monitors J's performance and she decides in each period whether to promote him. The return to the firm from promoting J depends on his innate ability. However, S does not know J's ability and must learn it over time. J also does not know his ability, but we assume that he maintains a dogmatic belief about it even though his direct interactions with S allow him to observe S's belief. If J believes his ability is high enough to make entrepreneurship profitable, he has not yet been promoted, and he doesn't expect to be promoted soon, he chooses to form a spinoff.

Although our model focuses on just one driving force behind employee spinoffs, it generates surprisingly diverse comparative statics effects. Consider, for example, the effect of greater uncertainty about employee ability. The model, which is cast in Bayesian terms, admits greater uncertainty at the time of hiring in the form of a noisier prior, and greater uncertainty at any subsequent point in time due to noisier signals. We show that noisier priors reduce the likelihood of spinoff formation, while noisier signals may work in either direction. As a second example, consider the role of overconfidence, measured by the excess of J's perception of his ability over its true value. If one holds perception fixed and raises overconfidence by reducing ability, the probability of spinoff formation unambiguously rises; in contrast, if one holds ability fixed while raising J's perception, the likelihood of spinoff formation may rise or fall. Our conclusion is that even narrowly cast models of employee entrepreneurship are consistent with empirical findings that vary markedly as the context changes.

2. The Model

Two employees, S and J, work together at a firm. J is hired as a junior, earns a wage w, and produces output v. The firm therefore earns surplus $v - w$ per period. J is also endowed with his ability for work as a senior employee, θ. If J is promoted, he produces output θ, earns a wage $w^p > w$, and produces surplus $\theta - w^p$ for the firm. Promotion is irreversible. At any point in time, J may leave to operate his own firm. If he does so, he earns per-period income of θ. However, creating a firm entails a fixed cost k. Upon hiring J, S knows only that θ is a random draw from a distribution F_0. The realization of θ must be learned over time by S, who uses information gained from supervising J to decide whether he merits promotion. J also does not know his ability but, in contrast to S, he holds a dogmatic belief that it is equal to $\hat{\theta}$.

The timing of moves is as follows. At the beginning of period t, J decides whether or not to form a spinoff. If J stays, S obtains a new signal about J's ability, updates her belief and chooses either to promote him or to wait at least another period. We assume that S's current belief is observable to J as a result of their interactions.

2.1 *Supervisor's Problem.* At time t, S believes J's ability is a random draw from the distribution $F_t(\theta)$, with density $f_t(\theta)$. In each period, S observes a signal, z, that has conditional density $g(z \mid \theta)$, so we can define the one-step-ahead Bayes map as

$$f_{t+1}(\theta) = \frac{g(z \mid \theta) f_t(\theta)}{\int g(z \mid \theta) f_t(\theta) d\theta}. \tag{1}$$

S must solve a dynamic programming problem in which the only link across periods is the evolution of beliefs. These types of problems have been analyzed in quite general terms by Easley and Kiefer (1988). Equation (1) can be written in terms of the prior and posterior distributions,

$$F_{t+1} = b(z, F_t), \tag{2}$$

where $b(z, F_t)$ is the transition from the prior belief F_t to the posterior belief F_{t+1} after observing signal z. Easley and Kiefer (1988) prove that, under quite general conditions, $b(z, F_t)$ is a well defined, continuous, and stationary function.

The subjective expected value to S of having J as a junior employee when her belief is F_t is given by the Bellman equation

$$V\left(F_t\right) = \max\left\{v - w + \beta(1-\lambda(F_t))\int\int V(b(z,F_t))g(z\mid\theta)\,dz\,dF_t(\theta),\right.$$
$$\left.\frac{1}{1-\beta}\left(\int \theta dF_t(\theta) - w^p\right)\right\}, \qquad (3)$$

where $\lambda(F_t)$ is S's subjective probability that J will depart at the beginning of the next period. If J departs, we assume the payoff to S is zero.

Stopping rules are beliefs, F^*, that satisfy

$$v - w + \beta(1-\lambda(F^*))\int\int V(b(z,F^*))g(z\mid\theta)\,dz\,dF^*(\theta) =$$
$$\frac{1}{1-\beta}\left(\int \theta dF^*(\theta) - w^p\right). \qquad (4)$$

In general, F^* is not uniquely defined by (4). Suppose, for example, that $F \sim N(\overline{\theta},\sigma^2)$. While the right hand side (RHS) of (4) is linear in θ and hence independent of σ^2, the left hand side (LHS) is convex in θ and depends jointly on the pair $\{\overline{\theta},\sigma^2\}$. Thus, (4) contains two unknowns and admits an infinite number of stopping distributions.[1]

2.2 *Junior's Problem.* A meaningful stopping problem exists only if J's dogmatic belief about his ability satisfies the inequalities

$$w < -k(1-\beta) + \hat{\theta} < w^p, \qquad (5)$$

so that founding a business is more attractive than wage employment when the agent knows he will never be promoted, and immediate promotion is more attractive than self-employment. Agents who believe their ability is too low to satisfy (5) never consider stopping and do nothing more than passively await a promotion decision by S, while agents with perceived ability too large to satisfy (5) never join the firm. To avoid tedious discussion of uninteresting cases, we shall assume that inequality (5) holds throughout.

Let $\mu^{\hat{\theta}}(F_{t-1})$ denote J's subjective probability that he will be promoted in period t, and let $W\left(\hat{\theta},F_{t-1}\right)$ be the value to J of not being promoted. The value function

[1] This does not imply that the problem is not well-behaved. For example, it is easy to show that, for any belief F there exists a unique $v(F)$ such that juniors producing $v{>}v(F)$ are not promoted, while those producing $v{<}v(F)$ are promoted.

W depends, *inter alia*, on both J's dogmatic belief and S's evolving belief. However, given the timing of moves, J makes decisions in each period based on S's belief in the previous period.

J's Bellman equation is

$$W\left(\hat{\theta}, F_{t-1}\right) = \max\left\{(1-\mu^{\hat{\theta}}(F_{t-1}))\left(w + \beta\int W(\hat{\theta}, b(z, F_{t-1}))g(z \mid \hat{\theta})dz\right)\right.$$
$$\left. + \mu^{\hat{\theta}}(F_{t-1})\frac{w^p}{1-\beta}, \quad -k + \frac{\hat{\theta}}{1-\beta}\right\}. \tag{6}$$

Stopping rules, F^{**}, satisfy

$$\frac{\mu^{\hat{\theta}}(F^{**})w^p - (\hat{\theta} - k(1-\beta))}{(1-\beta)(1-\mu^{\hat{\theta}}(F^{**}))} = w - \beta\int W(\hat{\theta}, b(z, F^{**}))g(z \mid \theta)dz\,. \tag{7}$$

Although, like F^*, F^{**} is not generally uniquely defined, the optimal policy has some straightforward properties. For example, for any belief F there exists a $k(F)$ such that J forms a spinoff if $k < k(F)$, and he otherwise remains with the firm.

2.3 *Normal Priors and Signals*

It will be unsurprising to readers familiar with Bayesian learning that we can make considerable progress if we assume the sequence of beliefs, $\{F_t\}_{t=0}^{\infty}$, belongs to the Normal conjugate family. Therefore, suppose that S's prior is that θ is drawn from a Normal distribution with zero mean and variance σ_θ^2, and that the signals are random draws from a Normal distribution with mean θ and variance σ_z^2. We continue to suppose that J maintains a dogmatic belief that his ability is $\hat{\theta}$. Let $\overline{z}_t$ denote the mean of the t signals observed up to period t. Using standard formulae [*e.g.,* DeGroot (1970, ch. 9)], S's posterior belief is Normal with mean

$$\overline{\theta}_t = \frac{\overline{z}_t t\sigma_\theta^2}{\sigma_z^2 + t\sigma_\theta^2}, \tag{8}$$

and variance

$$\sigma_t^2 = \frac{\sigma_\theta^2\sigma_z^2}{\sigma_z^2 + t\sigma_\theta^2}. \tag{9}$$

Because the variance of beliefs is a deterministic function of time, the pair $\{\overline{\theta}_t, t\}$ is a sufficient statistic for $F_t(\theta)$. The two stopping rules can now be written as

$$v - w + \beta(1 - \lambda_t(\overline{\theta}_t^*)) \int \int V(b(z, \overline{\theta}_t^*, t)) g(z \mid \theta)\, dz\, dF_t(\overline{\theta}_t^*) = \frac{\overline{\theta}_t^* - w^p}{1-\beta} \tag{10}$$

for promotion, and

$$\frac{(\hat{\theta} - k(1-\beta)) - \mu_t^{\hat{\theta}}(\overline{\theta}_t^{**}) w^p}{(1-\beta)(1-\mu_t^{\hat{\theta}}(\overline{\theta}_t^{**}))} = w + \beta \int W(\hat{\theta}, b(z, \overline{\theta}_t^{**}, t)) g(z \mid \hat{\theta}) dz \tag{11}$$

for spinoff formation. In (10) and (11), $g(z \mid \theta)$ is the density of a Normal random variable with mean θ and variance $\sigma_{z;}^2$, $\overline{\theta}_t^*$ is the critical value that induces promotion in period t, $\overline{\theta}_t^{**}$ is the critical value that induces J to form a spinoff in period t, and $b(z, \overline{\theta}_t, t)$ maps a Normal distribution with mean $\overline{\theta}_t$ and variance σ_t^2, into a Normal distribution with moments

$$\overline{\theta}_{t+1} = \frac{\overline{\theta}_t \sigma_z^2 + z_{t+1}\sigma_t^2}{\sigma_z^2 + \sigma_t^2}, \quad \text{and} \quad \sigma_{t+1}^2 = \frac{\sigma_z^2 \sigma_t^2}{\sigma_z^2 + \sigma_t^2}. \tag{12}$$

Note that, for S, $E^S[z_{t+1} \mid \overline{\theta}_t] = \overline{\theta}_t$, so (12) implies that $E[\overline{\theta}_{t+1} \mid \overline{\theta}_t] = \overline{\theta}_t$. This is, of course, just the law of iterated expectations. For J we have $E^J[z_{t+1} \mid \overline{\theta}_t] = E[z_{t+1}] = \hat{\theta}$. When S's current belief is less favorable than J's dogmatic belief, J is optimistic that future signals will induce an improvement in S's belief.

With this specification for beliefs, we can make the following statement about the stopping problems:

Proposition 1. (A) *As long as $\lambda_t(\overline{\theta}_t)$ does not decline too rapidly when $\overline{\theta}_t$ increases, there exists a unique stopping rule, $\overline{\theta}_t^*$, such that J is promoted the first time that $\overline{\theta}_t > \overline{\theta}_t^*$.* (B) *There exists a unique stopping rule, $\overline{\theta}_t^{**} < \overline{\theta}_t^*$, such that J forms a spinoff the first time that $\overline{\theta}_t < \overline{\theta}_t^{**}$.*

Proof. For part (A), suppose that $\lambda_t(\overline{\theta}_t) = \lambda_t$ is invariant to S's belief. Then, because increases in $\overline{\theta}$ affect V only through the possibility that J is promoted in the future, the derivative of the left-hand side (LHS) of (10) with respect to $\overline{\theta}$ can nowhere exceed $\beta(1-\lambda_t)$ times the derivative of the RHS evaluated at the same point. At the same time, if S is sufficiently confident that J has low ability (say, as $\overline{\theta}_t \to -\infty$), the LHS of (10) exceeds the RHS. These properties are illustrated in Figure 1, where **W** depicts the RHS and the curves labeled $\mathbf{V}_t$ and $\mathbf{V}_{t+j}$ plot the LHS for two different time periods. Because the option value of waiting declines as the posterior variance declines, **V** shifts down over time. As illustrated, each **V** intersects **W** once and, does so in such a way that an increase in t is associated with a reduction $\overline{\theta}_t^*$. This result is undermined if there are segments of $\lambda_t(\overline{\theta}_t)$ that fall sharply as $\overline{\theta}_t$ increases. In this case, the slope of **V** may exceed the slope of **W**, and if such a segment exists in just the right range, there may be multiple stopping values.

For part (B), note that the derivative of the LHS of (11) is

$$\frac{(\hat{\theta}-k(1-\beta))-w^p}{(1-\beta)(1-\mu_t^{\hat{\theta}}(\overline{\theta}_t^{**}))^2}\frac{\partial\mu_t^{\hat{\theta}}}{\partial\overline{\theta}_t}<0\,, \tag{13}$$

which is strictly negative because (i) the probability of promotion in the next period is strictly increasing in $\overline{\theta}_t$, and (ii) $w^p>(\hat{\theta}-k(1-\beta))$ [see eq. (5)]. Thus the LHS of (11) is strictly decreasing in $\overline{\theta}_t$, while the RHS is clearly increasing, and there is at most one intersection. As $\overline{\theta}_t\to-\infty$, $\mu_t^{\hat{\theta}}(\overline{\theta}_t)\to 0$; then the LHS of (11) approaches $(\hat{\theta}-k(1-\beta))/(1-\beta)$ and the RHS approaches $w/(1-\beta)$. At the other extreme, as $\overline{\theta}_t\to+\infty$, $\mu_t^{\hat{\theta}}(\overline{\theta}_t)\to 1$, the LHS approaches $-\infty$, and the RHS approaches $+\infty$. Hence, there is exactly one intersection (see Figure 2). To see that $\overline{\theta}_t^{**}<\overline{\theta}_t^*$, suppose not. Then for any belief such that J chooses to form a spinoff, S is also willing to promote J immediately. But, by (5), J prefers immediate promotion to spinoff formation, so he would not form a spinoff. •

Figure 3 provides a schematic representation of the two stopping boundaries. Learning is a straightforward process in our setting as signals and beliefs do not depend on actions. Consequently, $\overline{\theta}_t\to\theta$ at the rate $t^{-1/2}$. Two sample paths are drawn: one that converges on a low value of θ and leads to J forming a spinoff, and another that converges on a high value and leads to J's promotion. Two other possibilities are not drawn. In one, promotion is not optimal (because $\theta-w^p<v-w$) but J is promoted after misleading signals raise $\overline{\theta}_t$ above $\overline{\theta}_t^*$. In the other, misleading signals induce J to form a spinoff despite the fact that θ is large enough to merit promotion. We do not have a complete picture of the two stopping boundaries, but we can characterize some of their main properties.

• The option value of continuation declines as the precision of beliefs improves, so S's standard for promotion will become less demanding over time. Hence, S's boundary trends downwards. This decline gives the impression that there is promotion for time served, because the requirement for promotion is lower for

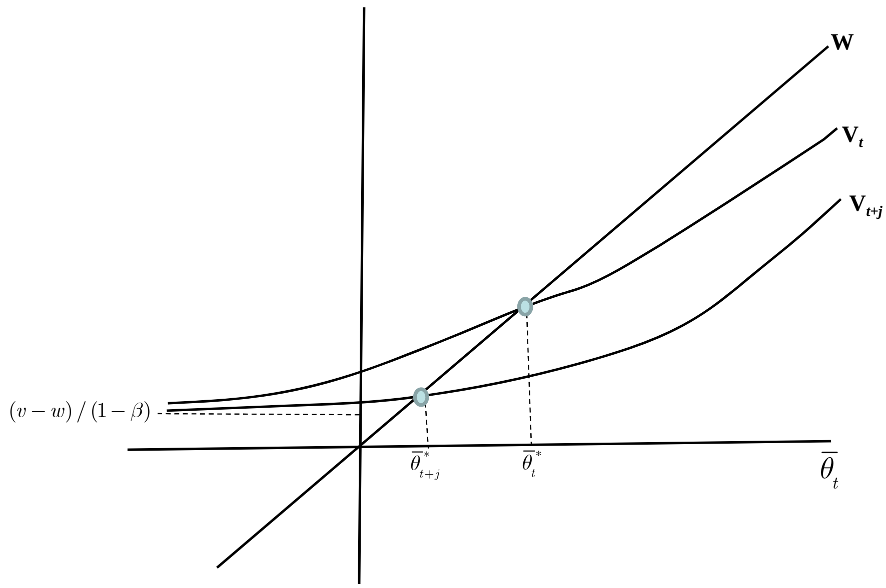

Figure 1

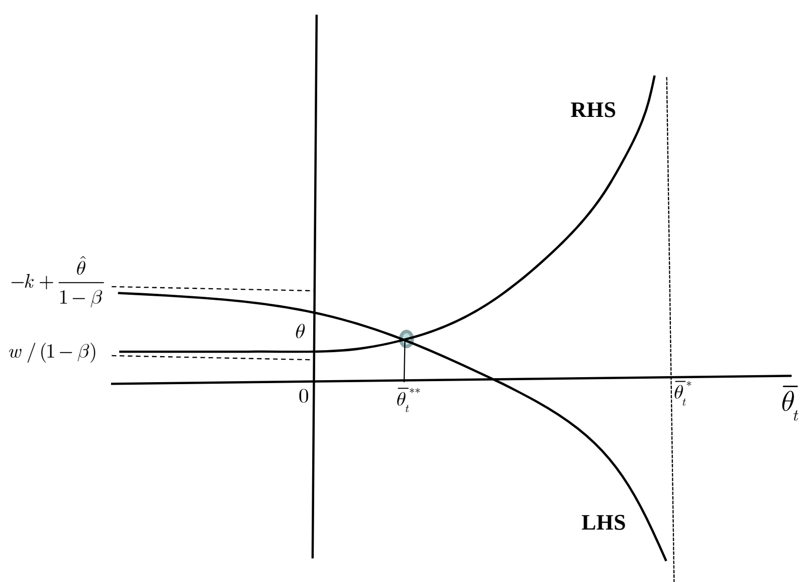

Figure 2

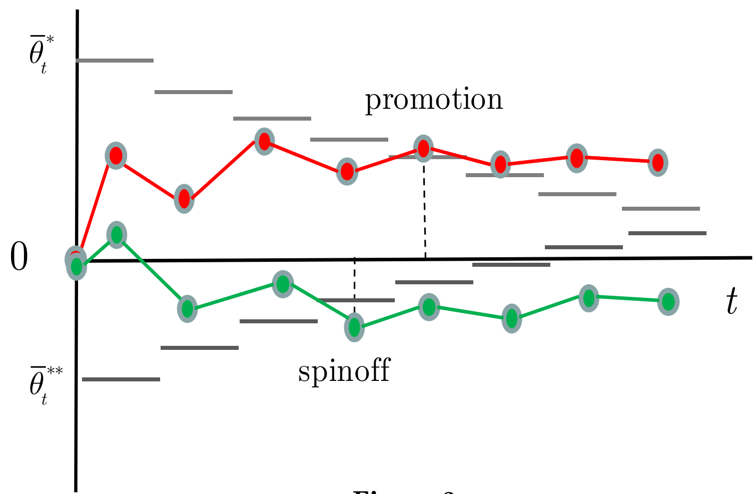

Figure 3

individuals who have been employed longer.[2] Equivalently, there is an appearance of fast-tracking because unusually good individuals are on average promoted quickly.

• Although we have drawn it as monotonically increasing, the trend for $\overline{\theta}_t^{**}$ is ambiguous. On the one hand, S's belief changes more slowly with the passage of time, and this makes J less willing to tolerate an unfavorable belief and remain with the firm. On the other hand, S's boundary is trending downwards, and this induces J to tolerate a less favorable belief. As a result, J's boundary may rise or fall. However, we can expect that the distance between the boundaries declines over time. Indeed, as $t \to \infty$, the boundaries must asymptotically coincide: when beliefs become fixed, because S has observed so many signals, J will not tolerate any belief that does not result in immediate promotion.

• In some cases, changes in parameter values have unambiguous effects on the stopping boundaries. We shall leave more detailed comparative statics analysis until later, and provide just one example here. An increase v has no direct bearing on how attractive spinoff formation is to J. However, it makes promotion less attractive to S (shifting $\overline{\theta}_t^*$ up), and this in turn induces J to be less willing to

[2] This is consistent with the perception among academics that the requirements for early tenure are higher than for tenure granted at the regular time.

tolerate an unfavorable belief about his ability. As a result, an increase in v shifts both $\overline{\theta}_t^*$ and $\overline{\theta}_t^{**}$ upwards. These changes make spinoff formation more likely and promotion less likely.

2.4 *Approximate Stopping Rules*

In related stopping problems, Jovanovic (1979), Thompson (2008), and Thompson and Chen (2011), have implemented an approximation to the distribution of stopping times by fixing critical values to their asymptotic levels. We adopt the same strategy here for S's problem; for J's problem, we will need to be a little more sophisticated.

We begin with S's problem. Let $\overline{\theta}_t^* = \lim_{t\to\infty} \overline{\theta}_t^*$ for all t, so that S chooses to promote J in the first period that $\overline{\theta} - w^p > v - w$. This approximation imposes a strong form of myopia on S, by ignoring the option value of waiting one more period to gather more information. Clearly, imposing myopia induces an underestimate of the stopping value. However, not taking into account the option value also means that we are ignoring S's subjective probability that J quits, and this serves to ameliorate the underestimate induced by myopia.

We analyze J's stopping decision using the one step look ahead (1sla) rule. That is, J compares the return from founding his own business today with waiting one more period to see if he is promoted and then forming a spinoff if he is not. The 1sla correctly recommends continuation when continuation is optimal, but it may recommend stopping when continuation is in fact optimal. Thus, the 1sla may provide an overestimate of $\overline{\theta}_t^{**}$.[3]

Under 1sla, J leaves the first time that

$$-k + \frac{\hat{\theta}}{1-\beta} \geq \mu_t^{\hat{\theta}}(\overline{\theta}_{t-1})\frac{w^p}{1-\beta} + (1-\mu_t^{\hat{\theta}}(\overline{\theta}_{t-1}))\left(w + \beta\left(-k + \frac{\hat{\theta}}{1-\beta}\right)\right), \tag{14}$$

or, upon rearranging, when

[3] The 1sla rule is known to be optimal for all monotone stopping problems [*e.g.*, Ferguson (2008, ch. 5)], However, J's problem is not monotone.

$$\mu_t^{\hat{\theta}}(\overline{\theta}_{t-1}) \leq \frac{\hat{\theta} - (w + (1-\beta)k)}{\frac{w^p - \beta\hat{\theta}}{1-\beta} - w + \beta k}. \tag{15}$$

The probability that J will be promoted, $\Pr\left\{\overline{\theta}_t > v + w^p - w\right\}$, is [from (12)] equal to the probability that the new signal, z_t, satisfies the inequality,

$$z_t \geq \frac{(v + w^p - w - \overline{\theta}_{t-1})(\sigma_z^2 + t\sigma_\theta^2)}{\sigma_\theta^2 \sigma_z} + \overline{\theta}_{t-1}, \tag{16}$$

where J believes that $z \sim N(\hat{\theta}, \sigma_z^2)$. The probability that inequality (15) is satisfied is therefore given by

$$\mu_t^{\hat{\theta}}(\overline{\theta}_{t-1}) = 1 - \Phi\left(\frac{(q - \overline{\theta}_{t-1})(\sigma_z^2 + t\sigma_\theta^2)}{\sigma_\theta^2 \sigma_z} + \frac{\left(\overline{\theta}_{t-1} - \hat{\theta}\right)}{\sigma_z}\right), \tag{17}$$

where $q = v + w^p - w$ and $\Phi(\bullet)$ is the distribution function of a standard Normal random variable. Using (17) in (15), θ_{t-1}^{**} is the solution to

$$\Phi\left(\frac{(q - \overline{\theta}_{t-1}^{**})(\sigma_z^2 + t\sigma_\theta^2)}{\sigma_\theta^2 \sigma_z} + \frac{\left(\overline{\theta}_{t-1}^{**} - \hat{\theta}\right)}{\sigma_z}\right) = \frac{w^p - \hat{\theta} + (1-\beta)k}{w^p - \beta\hat{\theta} + (1-\beta)(\beta k - w)}. \tag{18}$$

Note that the RHS of (18) lies in the unit interval for all agents with a meaningful stopping problem [see eq. (5)]. The LHS must remain constant as t advances. It is therefore easy to see that

$$\frac{d\theta_{t-1}^{**}}{dt} = \frac{q - \theta_{t-1}^{**}}{t-1}. \tag{19}$$

Because $v - w > \overline{\theta}_{t-1}^{**} - w^p$, (19) is positive for all $t > 1$. It then follows $\overline{\theta}_{t-1}^{**} \to q$ as $t \to \infty$ so, as claimed earlier, the boundaries for S and J converge.

2.5 *A Transformation to Gaussian Random Walks*

The sample paths of $\overline{\theta}_t$ are realizations of a stochastic process with normally distributed increments in each period, having mean $\overline{\overline{\theta}}_t = t\sigma_\theta^2\theta(\sigma_z^2 + t\sigma_\theta^2)^{-1}$ and variance $\sigma_{\overline{\theta},t}^2 = t\sigma_\theta^2\sigma_t^2(\sigma_z^2 + t\sigma_\theta^2)^{-1}$. In order to evaluate the comparative statics of stopping times, it is much easier to transform the problem so that the stochastic process is a Gaussian random walk. Hence, define

$$\xi_t = \left(\frac{\sigma_z^2 + t\sigma_\theta^2}{\sigma_z \sigma_\theta^2}\right)\overline{\theta}_t - \frac{t\theta}{\sigma_z}. \tag{20}$$

The variable ξ_t is normal with zero mean and variance t, while the increments to ξ_t are independent standard Normals. That is ξ_t is a Gaussian random walk.

The upper absorbing barrier for $\overline{\theta}_t$ is q. The corresponding barrier for ξ_t is obtained by replacing $\overline{\theta}_t$ in (20) with q. The transformed barrier for promotion is therefore

$$B^S(t) = \frac{\sigma_z q}{\sigma_\theta^2} - \left(\frac{\theta - q}{\sigma_z}\right)t. \tag{21}$$

To obtain the transformed barrier for spinoff formation, solve (20) for $\overline{\theta}_t$, back-date by one period and evaluate at $\overline{\theta}^{**}_{t-1}$,

$$\overline{\theta}^{**}_{t-1} = \frac{\sigma_\theta^2 \sigma_z \xi^{**}_{t-1} + \sigma_\theta^2 \theta(t-1)}{\sigma_z^2 + (t-1)\sigma_\theta^2}. \tag{22}$$

Finally, substitute this expression into (18):

$$\Phi\left(\frac{q\sigma_z}{\sigma_\theta^2} + \frac{\theta - \hat{\theta}}{\sigma_z} - \frac{(\theta - q)t}{\sigma_z} - \xi^{**}_{t-1}\right) = \frac{w^p - \hat{\theta} + (1-\beta)k}{w^p - \beta\hat{\theta} + (1-\beta)(\beta k - w)}. \tag{23}$$

The LHS of (23) must be constant over time. It then follows that

$$\frac{d\xi^{**}_{t-1}}{dt} = -\frac{\theta - q}{\sigma_z}. \tag{24}$$

Hence, the transformed barrier for firm creation is linear. Because J makes decisions at time t based on S's belief at time $t-1$, the barrier for spinoff formation given in (23) is defined only for $t \geq 1$. Hence, we can write

$$B^J(t) = \begin{cases} \psi_1(w, w^p, v, \hat{\theta}, k, \beta, \sigma_\theta^2, \sigma_z^2) - \left(\dfrac{\theta - q}{\sigma_z}\right)(t-1), & t = 1, 2, 3, \ldots \\ -\infty, & t < 1 \end{cases}, \tag{25}$$

where ψ_1 is the solution to (23) obtained after setting $t = 1$.

Figures 4 and 5 depict the resulting first passage problem. J is promoted if the Gaussian random walk, ξ_t, hits the upper barrier without previously hitting the lower barrier. J forms a spinoff if ξ_t hits the lower barrier without previously hitting the upper barrier. Because the barriers have the same slope, one of these events *must* eventually happen. The figures illustrate two qualitatively distinct cases. In Figure 4, $\theta < q$. S would not want to promote J if she knew J's ability, but she may mistakenly do so if the signals she observes are sufficiently misleading. In this case, both barriers have a positive slope. In Figure 5, $\theta > q$. S would immediately promote J if she knew his ability. In this case, the barriers have a negative slope.

As a result of the transformation of our problem into a stopping problem for a Gaussian random walk, all parameter changes have effects manifested only through shifting either or both of the linear stopping boundaries. It is readily apparent that anything that unambiguously shifts one or both barriers upwards makes spinoffs more likely and promotion less likely within any given finite time period. Similarly, anything that shifts one or both barrier downwards raises the probability of promotion while reducing the probability of a spinoff. Any parameter change that moves the barriers in opposite directions has an ambiguous effect. To see why, suppose that both barriers shifted inward. Although it is more likely that any sample path for ξ_t hits the upper barrier, it may be harder for the sample path to hit the upper barrier without first hitting the lower barrier. Hence when both barriers shift inward, the probability that J is promoted may rise or fall.

Table 1 summarizes how parameter changes move the two barriers. $B^S(\theta,t)$ shifts downwards in response to increases in θ, σ_θ^2, and w, and upwards in response to increases in v and w^p; $B^J(\theta,t)$ is decreasing in k, θ, σ_θ^2, and w, and increasing in v and w^p. The combined effects of the shifts in both barriers are summarized in columns (3) and (4) of Table 1 and in Proposition 2.

PROPOSITION 2. (1) *The probability that J is promoted by any time t is:* (i) *increasing in θ, σ_θ^2, w, and k, and* (ii) *decreasing in v and β.* (2) *The probability that J forms a spinoff by any time t is:* (i) *decreasing in θ, σ_θ^2, w, and k, and* (ii) *increasing in v and β.*

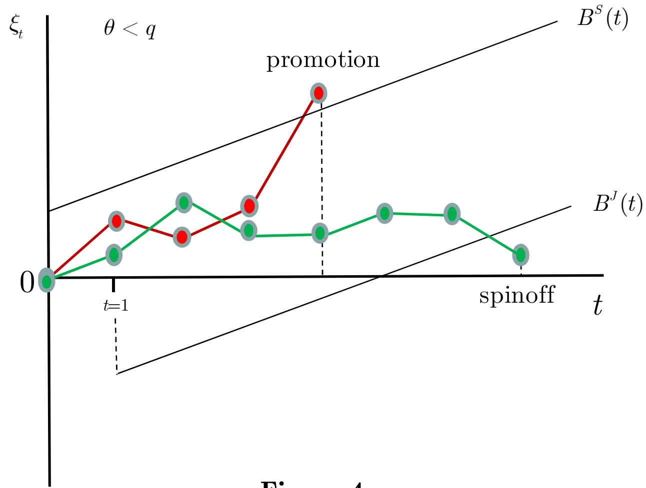

Figure 4

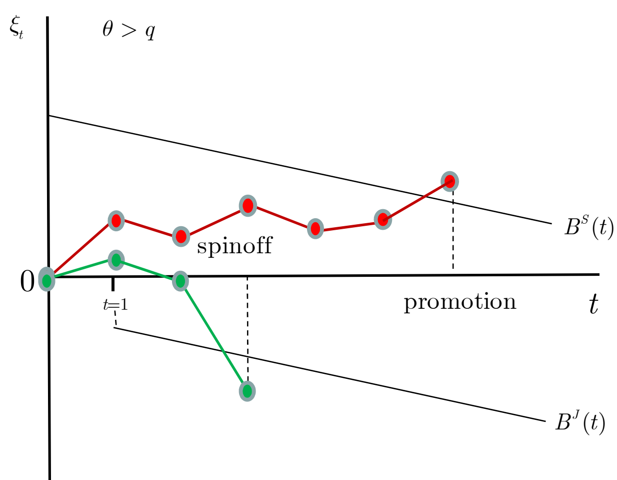

Figure 5

Table 1

Increasing parameter	Shift in $B^S(\theta,t)$	Shift in $B^J(\theta,t)$	Change in Probability by time t	
			Promotion	Spinoff
	(1)	(2)	(3)	(4)
A. Unambiguous Effects				
θ	Down	0* / Down**	+	−
σ_θ^2	Down	Down	+	−
k	0	Down	+	−
w	Down	Down	+	−
β	0	Up	−	+
v	Up	Up	−	+
B. Unsigned Effects				
$\hat{\theta}$	0	?	?	?
σ_z^2	?	?	?	?
w^p	Up	?	?	?

* For $t = 1$.** For $t > 1$.

These effects of parameter changes on the promotion and spinoff probabilities are straightforward and, to avoid tedious repetition, we discuss just two results:

(1) *Uncertainty about ability.* An increase in the prior variance of S's belief, σ_θ^2, makes S more responsive to new information, and hence more likely to promote S by any given time period. This increased likelihood of promotion induces J to be more tolerant of unfavorable beliefs and hence less likely to form a spinoff. In contrast, an increase in σ_z^2 has two countervailing effects on beliefs: on the one hand, the greater variability of signals increases the possibility of beliefs that diverge widely from the true value; on the other hand, knowing that the signals are noisier makes S less responsive to new information. These countervailing forces induce ambiguous impacts on both boundaries.

(2) *Overconfidence*. Let $\hat{\theta} - \theta$ measure J's overconfidence about his ability. An increase in overconfidence can be brought about either by an increase in J's sub-

jective evaluation of his ability or by a reduction in his true ability. A reduction in ability makes it more likely that S will develop a poor opinion of *J,* unambiguously making promotion less likely and spinoff formation more likely. An increase in $\hat{\theta}$, in contrast, has no effect on $B^S(t)$ and may shift $B^J(t)$ in either direction. The ambiguous impact on the spinoff barrier arises because an increase in $\hat{\theta}$ increases the desirability of both waiting for promotion and of spinoff formation: for any given ability, a more confident J expects poor opinions to be corrected quickly while at the same time he expects greater earnings from running his own company. As a result, the overall effect of increased overconfidence is unsigned, depending in large part on whether variations in overconfidence are driven by variations in perceptions or variations in actual ability.

2.6 *Continuous Time Density for First Passage*

It is common to analyze first passage problems by passing to continuous time. Doing so is of somewhat limited value here, because of the non-trivial lag between the time information is obtained and used by S and the information used by J at the time he makes decisions. Nonetheless, it is instructive to derive an approximation to the stopping times as follows. The continuous time stochastic process that gives rise to the same distribution as ξ_t at t=0, 1, 2, . . . , is a standard zero-drift Wiener process, $\xi(t)$, with initial condition $\xi(0) = 0$. Because spinoff formation is not admitted at any time $t < 1$, it is appropriate to disallow promotion during this same period.

Under these assumptions, the density of first passage times can be written in closed form:

THEOREM 1. *Let* $a = B^S(1) - v$, $b = B^J(1) - v$, *and* $\gamma = (\theta - q) / \sigma_z$. *The density of first passage times is*

$$f(t) = \begin{cases} 0, & t < 1 \\ 1 - \Big(\Phi(B^S(1)) - \Phi(B^J(1))\Big), & t = 1 \\ \displaystyle\int_{B^J(1)}^{B^S(1)} f_{ab}(t \mid v)\, d\Phi(v), & t > 1 \end{cases}$$

where $\Phi(s)$ *is the distribution of a standard Normal random variable, and*

$$\tilde{f}_{ab}(t \mid v) = \frac{\pi}{(a-b)^2} \sum_{n=1}^{\infty} n(-1)^{n+1} e^{\frac{(1-t)}{2}\left(\gamma^2 + \frac{n^2\pi^2}{(a-b)^2}\right)} \left(e^{\gamma b} \sin\left(\frac{nb\pi}{a-b}\right) - e^{\gamma ha} \sin\left(\frac{na\pi}{a-b}\right)\right).$$

The conditional density, $f_{ab}(t \mid v)$,[4] is plotted in Figure 6 for a case in which promotion is *ex post* optimal from the perspective of S (*i.e.*, where $\theta > q$), and for three values of $\xi(1)$. The corresponding unconditional density, $f(t)$, is in bold. $\tilde{f}_{ab}(t \mid v)$ may be unimodal or bimodal. The bimodal case arises when $\xi(1)$ is close to the spinoff barrier at $t = 1$. In the bimodal case, the first mode corresponds almost entirely to agents forming spinoffs in response to initially unfavorable beliefs held by S; the second mode consists almost entirely of agents being promoted. The unimodal case emerges when $\xi(1)$ is further from the spinoff barrier at $t = 1$. In this case, spinoffs are vanishingly rare and first passage events are dominated by promotions. When spinoffs are not *ex post* optimal for S, (*i.e.*, where $\theta < q$), it is again possible to obtain both unimodal and bimodal densities. In this second bimodal case, the first mode consists almost entirely of mistaken promotions, while the second mode consists almost entirely of spinoffs.

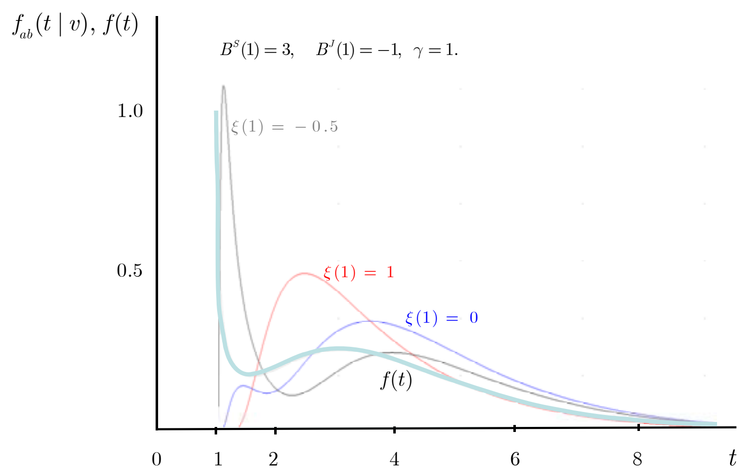

Figure 6

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[4] This was first derived by Darling and Siegert (1953:632-634). Dominé (1996) provides a different derivation and corrects a misprint in the original.

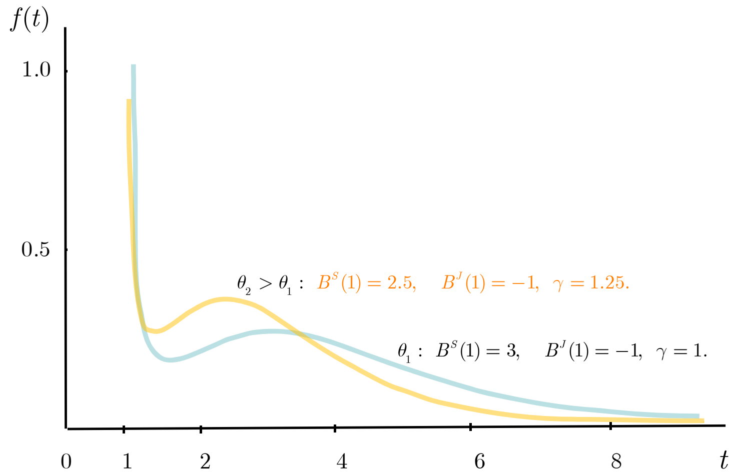

Figure 7

The unconditional density, $f(t)$, consists of a mass point at time $t = 1$, and thereafter is obtained by taking expectations over all the conditional densities, $f_{ab}(t)$. Figure 7 illustrates for two values of $J's$ ability, θ, both of which would induce immediate promotion if it were known.[5] An increase in J's ability leads to a reduction in very early first passages, while the second mode appears earlier. The decline in very early first passage times is the result of a decline in the number of spinoffs, while the movement in the second mode arises because promotions take place more quickly.

3. Conclusions

In this paper, we developed a simple dynamic model of promotion and spinoff formation in the presence of uncertainty about employee ability. A supervisor must learn over time about the employee's aptitude for work at a more senior level, while the employee trades off the benefits of waiting for promotion against immediate departure to form a spinoff. By means of a number of approximations to the pair of interrelated optimal stopping problems that our model gives rise to,

[5] Increases in θ shift the promotion barrier downward, and increase [decrease] the absolute value of the slope of both barriers when the slopes are initially negative [positive]

we are able to characterize the effects of a variety of parameter changes on the likelihood and timing of promotion and spinoff formation.

In our model, the time of promotion is variable, the wage is fixed until promotion, there is no dismissal, and entrepreneurship is the only outside option available to the employee. However, in most settings, employment at another firm is the most likely outside option; in many cases, especially where annual bonuses are paid, the wage adjusts in every period; in others, employees face up-or-out evaluations after a fixed period of tenure as a junior employee; in yet others, early promotion is rare but early dismissal can be common. Each of these deviations from the context we have modeled may have significant consequences for the rate, timing and comparative statics of both promotion and entrepreneurship. As a result, empirical work conducted in different settings can be expected to yield mixed results, as has been the case up to now.

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