Entrepreneurs, Managers, and the Organization of Work

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This paper presents an explicit model of the entrepreneur's role in organizing the work undertaken by employees. The model assumes that agents vary in their ability to carry out this task, and so in one sense the model is a special case of Lucas' [*Bell Journal of Economics*, 9, pp. 508-523, (1978)] span of control framework. However, the model also relates ability to risk. As in Kihlstrom and Laffont [*Journal of Political Economy*, 87, pp. 719-748 (1979)], the entrepreneur bears all risk. But while Kihlstrom and Laffont assume the entrepreneur bears all risk and show that as a consequence the least risk averse become entrepreneurs, in the present model all agents are risk neutral but entrepreneurs bear all risk as an equilibrium outcome. The model is used to study the consequences of firm growth driven by entrepreneurial learning or by rising demand.

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1. Introduction

Baumol's (1968) well-known observation that the theoretical firm is "entrepreneurless" was accompanied by an enormous degree of pessimism that the state of affairs would ever change. Almost forty years later, Bianchi and Henreckson (2005: 370) were no more optimistic: "the entrepreneur 'lacks operational definition' and is too elusive to ever fit into the neoclassical model." This continued pessimism is surprising in view of the significant theorizing about the entrepreneur over the last few decades. We have learned much from the literature spawned by the seminal contributions of Lucas (1978), Kihlstrom and Laffont (1979), and Holmes and Schmitz (1990), each of which focuses on a different facet of entrepreneurship and management. In one sense, however, these pioneering papers are very much in keeping with Baumol's pessimism. Each is quite vague about what exactly it is that entrepreneurs do. Lucas is most explicit: his model "does not say anything about the nature of the tasks performed by managers, other than that whatever managers do, some do it better than others." [p. 511].

This paper analyzes a production technology in which I try to be a little more specific about some of what entrepreneurs do. The essence of the production technology is as follows. An entrepreneur hires n workers. Each worker carries out exactly one unit of work per unit of time, so that n units of work are done in the firm. The value of this work, however, depends entirely on the decisions of the entrepreneur.[1] Each unit of work has a well-defined direction. The entrepreneur must decide the direction he would like this work to take, and he is responsible for organizing workers in this direction. Conditional on the intended direction, the actual direction of individual workers depends on the success with which the entrepreneur organizes his employees; the less directed are the employees, the less progress they make in the intended direction.

Individuals are assumed to vary in their ability to organize workers. The model predicts that the most able become entrepreneurs, while the least able become employees. Among entrepreneurs, the most able manage the largest firms and earn the most after paying the wage bill. Because better entrepreneurs manage more workers, in equilibrium the productivity of workers is the same in large and small firms. The model is obviously a version of Lucas' (1978) span of control

1. For example, a typing pool produces a fixed number of words per day regardless of the boss's identity. How valuable this typing is, however, depends on the abilities of the boss.

framework,[2] but it also bears some relation to the work of Kihlstrom and Laffont (1979) on risk bearing and of Holmes and Schmitz (1990) on business transfers.

In Kihlstrom and Laffont, entrepreneurs are assumed to earn stochastic profits that are the residual earnings after fixed payments to factors. In their framework, some individuals are better suited to entrepreneurship because they are less risk averse.[3] Entrepreneurs also bear all risk in the present model, even though all agents are risk neutral. The expected level and variability of firm output depends only upon the ability of, and effort expended by, the entrepreneur. Any reward scheme that does not require the entrepreneur to bear all risk induces insufficient effort, and is therefore inefficient. Thus, in contrast to Kihlstrom and Laffont, in which risk-bearing is a maintained assumption but is not efficient relative to arrangements in which employees share some of the risk, this paper produces risk-bearing as a consequence of the task of organization.

The model predicts volatile earnings for entrepreneurs but not for wage workers and, among entrepreneurs, a positive cross-sectional correlation between mean earnings and their volatility. These predictions are also consistent with a framework in which attitudes to risk determine occupational choice. However, after showing that the degree of risk aversion required to explain occupational choice in PSID data far exceeds conventional estimates, Rosen and Willen (2002) conclude that risk attitudes cannot be a major determinant of the decision to become self-employed. As previous empirical work imposes no *a priori* constraints on the distribution of abilities, the present model provides an avenue to explain Rosen and Willen's results.

Section 2 analyzes the model in a static setting. Section 3 introduces dynamics of two types. In the first, I assume that entrepreneurs (and possibly employees) can

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2. In Lucas, individuals vary in their talent for management, and greater talent increases the output that can be obtained from managing a given set of inputs. The same mechanism can be found in Rosen (1978, 1982), Calvo and Wellisz (1980), and Irigoyen (2002). Murphy, Shliefer and Vishny (1991) explore the implications of the model for economic development.

3. See also Kanbur (1979) and Blanchflower and Oswald (1998). Cressy (2000) and Hopenhayn and Vereshchagina (2003) point out that variations in wealth or borrowing constraints can induce different degrees of risk aversion among agents with identical preferences. Rigotti, Ryan and Vaithianathan (2005) have extended their framework to tolerance of ambiguity.

learn over time to organize better. In the second, I assume that demand grows over time. In both cases, firms grow as they age, and entrepreneurial earnings rise. But other features of firm evolution depend upon the source of growth. When growth is driven by learning, measured labor productivity rises if price falls in equilibrium, and remains constant if price does not fall. In contrast, firm and industry growth driven by rising demand induces a decline in productivity: price increases induce entrepreneurs with unchanged ability to increase their payrolls to exploit the rising demand, but they do so at the cost of diluting their control over workers. More substantively, demand driven growth (but not growth driven by learning) may induce some entrepreneurs to sell their business to agents with better organizational ability. The model predicts that businesses that are transferred are above average size and earn greater profits, as is the case in the two models of business transfers developed by Holmes and Schmitz (1990, 1995).

It is perhaps worth closing this section with a brief comment on terminology. I have throughout this introduction referred to agents as either workers or entrepreneurs, while I have also clearly limited attention to a single, management, function of the entrepreneur. Indeed, some of what follows is equally applicable to managers, although managers do not usually have claims to the entire residual profits of the firm after payment of fixed wages. For good or bad, this interchangeability of the terms entrepreneur and manager has a long history that dates back to Say (1880), and Franco (2005) has noted that it has persisted in much of the industrial organization literature that has followed Lucas (1978). Of course, some observers may decide that this focus on a management function of entrepreneurs ignores the most distinctive characteristics of entrepreneurship. Notwithstanding these concerns, I shall use the term entrepreneur throughout, although some readers may prefer the term small business owner.[4]

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4. Bianchi and Henrekson (2005:355), for example, argue that "entrepreneurship is not only management" and it is conducted in a setting replete with Knightian uncertainty, in which risk cannot even be quantified. However, it is not obvious to me that the degree and nature of risk provides an especially useful distinction between entrepreneurs and managers. In our datasets, entrepreneurship is most frequently measured by self-employment, and I am unconvinced that the CEO of, say, Ford, faces risk that is more quantifiable than that faced by the entrepreneur setting up a landscaping business.

2. The Model

An entrepreneur employs n workers, each of whom does one unit of work per period. How productive this work is depends upon how well each worker's activity is aligned with the entrepreneur's strategy. The degree of alignment is a random variable, but the expected distance between the strategy and the directed activity depends upon the entrepreneur's ability in direction and coordination. Figure 1 illustrates the implementation of this idea for $n = 3$. The firm's choice of strategy is represented by the slope of the line **OS**. Each of three workers carries out one unit of work, indicated by the lines x_1, x_2 and x_3. The contribution of this work toward meeting the firm's goals depends upon the relation between the direction of strategy and the direction of the worker's effort. For example, the contribution of worker 1 is found by connecting a line from **a** to **OS**, drawn normal to **OS**, and measuring the distance along **OS** from the origin to the point y_1. Clearly, if worker effort and firm goals are perfectly aligned, $y_1 = 1$. I assume that no worker contributes a negative amount, so the angle between x_i, and **OS** cannot exceed $\pi / 2$ radians.

Worker contributions are additive, so we can evaluate the contribution of all three workers by the simple geometric expedient of moving the origin of x_2 to **a**, and the origin of x_3 to **b**. The total contribution toward the firm's strategy is given by Y, measured along **OS**. The transformation of Y into output or profit depends upon the entrepreneur's choice of strategy.[5] In this paper, I put the question of strategy choice to one side, and take the liberty of interpreting Y as output. It is therefore convenient to rotate the coordinates to suppress the strategy question, as shown in Figure 2. As each of the line segments x_i has unit length, the contribution of worker i to output is given by $y_i = \cos(\theta_i)$. The constraint $-\pi / 2 \leq \theta_i \leq \pi / 2 \, \forall i$ implies $y_i \in [0,1] \, \forall i$.[6]

5. Readers will recognize a close affinity to the geometric analysis of forces applied to an object in classical mechanics. One difference, however, is that the effective force in classical mechanics is indicated by the length of a line connecting the origin to point **c**.

6. The assumption that the contribution to output of worker i's actions does not depend on the activity of worker j contrasts sharply with the production technology in recent models of coordination. In Dessein and Santos (2006), for example, the contribution to output of a worker undertaking task i depends upon the actions of workers undertaking all other tasks.

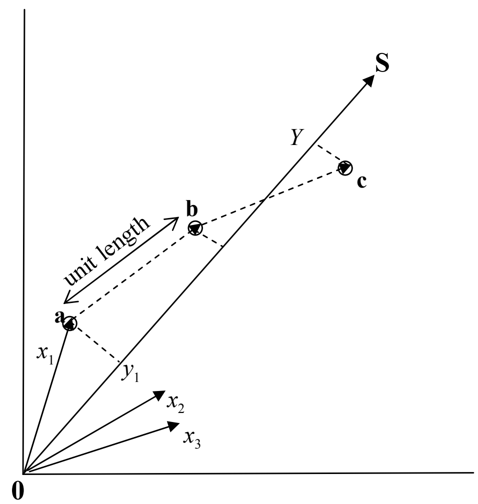

Figure 1. The coordination problem.

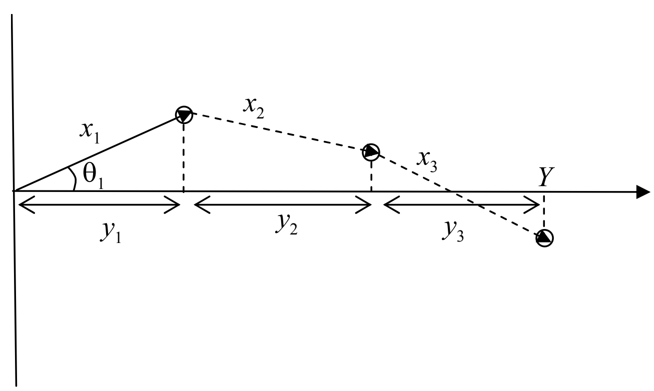

Figure 2. The coordination problem after rotation of coordinates.

Workers operate under imperfect direction and coordination; they do what they are told but what they are told to do may not be perfectly aligned with the firm's strategy. Assume, therefore, that each θ_i is a random draw from a distribution F with support on the interval $[-\pi/2, \pi/2]$. Note that the θ_i are not choices of the workers; they are the consequences of directions from the entrepreneur, but deviations from the optimal choice of zero should be thought of as mistakes. It is convenient to separate the mean and variance of the θ_i, so I assume that F is truncated normal with density

$$f_\theta(\theta) = \frac{e^{-\theta^2/2\sigma^2}}{\sigma\sqrt{2\pi}\,\mathrm{erf}\left(\pi/\left(2\sqrt{2}\sigma\right)\right)}, \tag{1}$$

for $\theta \in [-\pi/2, \pi/2]$, and zero otherwise. Recall that σ^2 is *not* the variance of θ. With symmetric truncation around a zero mean, the variance is

$$\mathrm{var}(\theta) = \sigma^2 - \frac{\sqrt{2\pi}e^{-\pi^2/8\sigma^2}}{2\,\mathrm{erf}\left(\pi/\left(2\sqrt{2}\sigma\right)\right)}, \tag{2}$$

which is nonetheless increasing in σ. The parameter σ is our measure of the degree of coordination effected by the entrepreneur. Perfect coordination is attained whenever $\sigma = 0$. At the other extreme, as $\sigma \to \infty$, $f(\theta)$ attains the uniform distribution on $[-\pi/2, \pi/2]$, so that $\lim_{\sigma\to\infty} \mathrm{var}(\theta) = \pi^2/12$. However, it is clear from Figure 2 that we are primarily interested in the distribution of the contributions, y_i. A simple transformation of (1) yields the required density:

$$f_y(y) = \frac{\sqrt{2}e^{-\left(\cos^{-1}(y)\right)^2/2\sigma^2}}{\sigma\sqrt{\pi\left(1-y^2\right)}\,\mathrm{erf}\left(\pi/\left(2\sqrt{2}\sigma\right)\right)} \tag{3}$$

for $y \in [0, 1]$, and zero otherwise. The expectation, $E[y;\sigma]$ is strictly decreasing in σ, with $E[y;0] = 1$ and $\lim_{\sigma\to\infty} E[y;\sigma] = 2/\pi$. Even the worst coordinators can own firms producing positive output, but they will produce substantially less than the best. Similarly, $\mathrm{var}(y;\sigma)$ is strictly increasing in σ, with $\mathrm{var}(y;0) = 0$ and $\lim_{\sigma\to\infty} \mathrm{var}(y;\sigma) = \tfrac{1}{2} - \tfrac{4}{\pi^2}$.

If σ did not vary with the number of workers to be directed, expected output for a firm with n workers would be $E[Y;\sigma,n]=nE[y;\sigma]$, indicating constant returns to scale. Consequently, the best entrepreneur (with the lowest σ) would employ all the workers and produce all the output. But of course, a single entrepreneur will rapidly become thinly stretched as n rises, and his ability to direct individual workers must consequently decline as a firm grows. A simple parameterization that I will use here sets $\sigma(n)=n^{\beta}/s$ with both β and s strictly positive and finite; all potential entrepreneurs face the same β, while s is larger for the better ones. The extent of coordination is a function of both the ability of the entrepreneur and the effort he expends. In this subsection I treat s as an exogenous characteristic of the entrepreneur and so refer to it as an index of ability. In the next subsection, I introduce entrepreneurial effort.

Let p denote the product price, and normalize wages to unity. Assuming further that the opportunity cost for the entrepreneur is his foregone wage, the expected net earnings of the entrepreneur, $E[w_e]$, are

$$E\left[w_e;s,p)\right]=E\left[\pi;s)\right]-1=$$

$$\max_n\left[np\int_0^1\frac{sy\sqrt{2}e^{-\left(\cos^{-1}(y)\right)^2/2\left(n^{\beta}/s\right)^2}}{n^{\beta}\sqrt{\pi\left(1-y^2\right)}\,\mathrm{erf}\left(\pi s/\left(2\sqrt{2}n^{\beta}\right)\right)}\,dy-(n+1)\right]. \tag{4}$$

For the time being, p will be treated as an exogenous parameter, as though this were an industry in a small country open to trade in the good. However, there is an upper bound to the price. When $s\to 0$, revenues approach $2np/\pi$, so $E[w_e]$ is strictly positive for arbitrarily bad entrepreneurs managing arbitrarily large firms whenever $p>\pi/2$. Thus, it is necessary to assume that $p\le\pi/2$.

2.1 Ability and Firm Size

The first result states that the best entrepreneurs operate the largest firms, which replicates a central result in Lucas (1978):

PROPOSITION 1. *The optimal firm size satisfies* $n(s)=\varphi(p,\beta)s^{1/\beta}$.

PROOF. Differentiate (4) with respect to n and substitute $\varphi=n/s^{1/\beta}$. Equating the resulting expression to zero yields the first order condition for firm size,

$$p\int_0^1 \left[\frac{\sqrt{\pi}\beta e^{-\pi^2/8\varphi^{2/\beta}}}{\sqrt{2}\,\mathrm{erf}\left(\pi/2\sqrt{2}\varphi^{1/\beta}\right)} + \frac{\beta}{\varphi^{1/\beta}}\left(\cos^{-1}(y)\right)^2 + \varphi^{1/\beta}(1-\beta)\right]\frac{ye^{-\left(\cos^{-1}(y)\right)^2/2\varphi^{2/\beta}}}{\sqrt{1-y^2}}\,dy$$

$$= \frac{\varphi^{2/\beta}\sqrt{\pi}}{\sqrt{2}}\,\mathrm{erf}\left(\frac{\pi}{2\sqrt{2}\varphi^{1/\beta}}\right), \qquad (5)$$

which depends on s and n only through, φ. For any given p and β, there is a unique solution to (5), $\varphi(p,\beta)$, with $\partial\varphi/\partial p > 0$ and $\partial\varphi/\partial\beta < 0$. This solution is not finite if $\sigma(n)$ does not rise sufficiently rapidly with firm size. That is, there exists a $\beta^*(p) < 1$ such that $\varphi(p,\beta) < \infty\ \forall\beta > \beta^*(p)$ and $\varphi(p,\beta) \to \infty$ $\forall\beta \le \beta^*(p)$.[7] In the former [latter] case, it is easy to verify that $\varphi(p,\beta)$ defines a maximum [supremum] of (4). •

It will be no surprise that the integral in (4) cannot be evaluated analytically. To provide some additional insight into the model, it is therefore useful to supplement the main propositions with some numerical evaluations. Figure 3 illustrates the relationship between $E[w_e]$ and n for several values of s for the case $\beta = 1$ and $p = \pi/2$. As Proposition 1 established, the size of firm that maximizes the entrepreneur's expected income is clearly increasing in *s*. Expected profit is higher for larger firms, and this is also a general result: if it were not, high-ability entrepreneurs (who operate the larger firms) could match the firm size of low-ability entrepreneurs and still outperform them. One's intuition should be that n rises more [less] rapidly with ability when β is small [large]. However, it is only possible to solve for $\varphi(p,\beta)$ numerically; doing so for selected values of β yields the plots shown in Figure 4, which are consistent with intuition.

2.2 Labor Productivity and Firm Size

How do unit output measures, such as labor productivity, vary with firm size? The answer is a straightforward consequence of Proposition 1. As $n(s) = \varphi(\beta)s^{1/\beta}$, it immediately follows that $\sigma = \varphi(p,\beta)^{\beta}$ is constant. Hence,

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7. When $\beta = 1$ and $p = \pi/2$, $\varphi(p,\beta) \approx 0.56$. In the remainder of the paper, I assume that $\beta > \beta^*$.

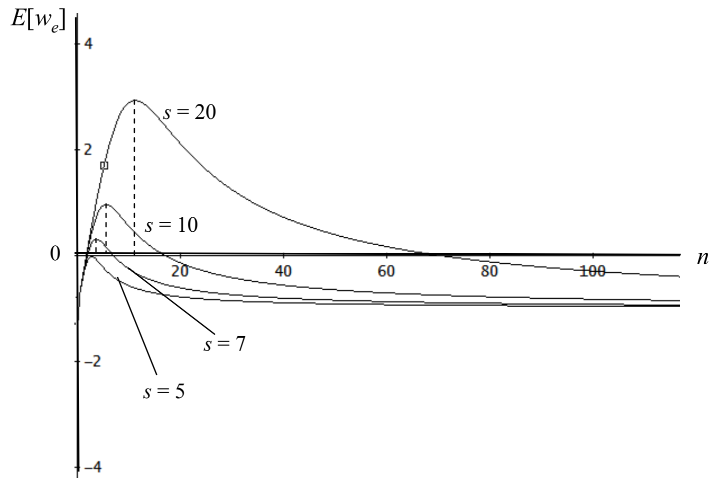

Figure 3. Entrepreneurial ability and the size of firms (I).

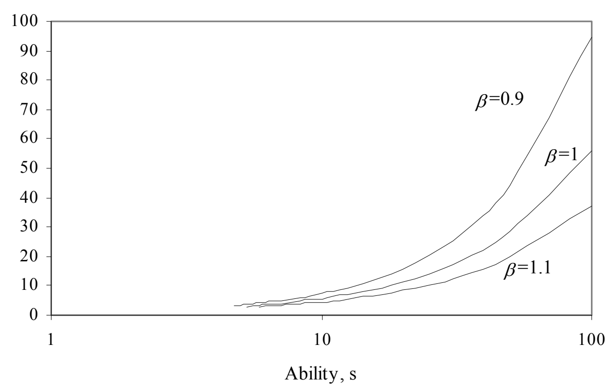

Figure 4. Entrepreneurial ability and the size of firms (II).

using (3), expected productivity is invariant to firm size and, therefore, to entrepreneurial ability.[8] When $\beta = 1$ and $p = \pi/2$ expected productivity is equal to approximately 86 percent of the productivity that would prevail under perfect coordination.[9]

PROPOSITION 2. *For any given price, expected labor productivity is invariant to firm size.*

The invariance of expected productivity has important consequences for traditional empirical research that measures only hired factors of production. First, the researcher would infer that the industry operates under a constant returns to scale production technology. This inference leads the researcher to conclude that firm size is indeterminate, which conclusion is supported directly by the observed variation in firm size [cf. Rosen (1982, p. 316)]. The researcher further notes the existence of residual profits after paying the wage bill and concludes that workers are being paid less than their marginal product, presumably due to some sort of market imperfection [cf. Bhaskar, Manning, and To (2002)]. None of these conclusions would be correct.

In a similar vein, the variance of individual contributions to output is invariant to firm size. The following proposition is an immediate consequence:

PROPOSITION 3. *The variance of firm output increases with firm size at the rate n. The variance of labor productivity declines with increasing employment at the rate $1/n$.*

The first statement is a trivially-familiar characteristic of firm size distributions: split a sample into large and small firms, and the within-group variance of the large firms is greater than the within-group variance of the small firms.[10] The relationship between firm size and productivity is perhaps less readily apparent. Bartelsman and Dhrymes (1998) study plant productivity in a substantial sample

8. The invariance of productivity to firm size is also a feature of Lucas (1978).

9. That is when $\sigma \equiv 0 \forall n$, in which case firm size is indeterminate. Expected productivity is somewhat higher [lower] when $\beta > [<] 1$, but numerical evaluations suggest productivity is not especially sensitive to variations in β. For example, a ten percent change in β around β=1 is associated with a two percent change in expected productivity.

10. Because firm age and size are positively correlated, it is also the case that the variance of firm size increases with cohort age [e.g. Cabral and Mata (2003)].

of large US plants. They construct two measures of plant productivity: one using an econometric approach which attributes factor shares to stochastic deviations from optimization, the other using a Solow residual approach that attributes to factor remuneration variations in output that may arise from randomness in the production process. The former concept of productivity is more consistent with the present model, and we summarize their results from that approach. They find: (i) larger plants are not more productive than smaller plants;[11] (ii) productivity is not correlated with age; and (iii) larger plants are less likely to move up or down in relative productivity rankings.[12] The model's predictions about productivity are consistent with this evidence.

2.3 Factor Shares

In this subsection, I replicate another result from Lucas (1978). Using the optimal employment level, $n = \phi(p, \beta) s^{1/\beta}$, to eliminate s from (4) yields an expression showing that residual profits after wages are proportional to n. The key result follows immediately:

PROPOSITION 4. *The entrepreneurial rent share,* $E[\pi]/\big(E[\pi] + n\big)$, *is independent of firm size.*

Of course, as n is an increasing function of ability, the model allows for possibly significant variation in entrepreneurs' incomes.

2.4 Entrepreneurial Risk Bearing

Kihlstrom and Laffont (1979) have argued that a key function of the entrepreneur is to bear risk. They analyze a model in which inputs are paid fixed wages and the entrepreneur receives the residual profit, which is stochastic. But why is this particular arrangement appropriate? In the present model, it can be justified

11. This is their only finding in the list that is not robust to methodology. With productivity measured by Solow residuals, productivity rises with size. Bartelsman and Dhrymes (1998) note that they have no explanations for the difference in results. Outlon (1998) tentatively concludes that large British firms may be *less* productive than small firms, while Lentz and Mortenson (2005) observe no relationship between employment and labor productivity among Danish firms.

12. Palangkaraya, Stierwald and Yong (2005) replicated this result for large Australian firms.

on efficiency grounds. Workers accomplish the same amount of work regardless of where they work; the risk associated with their contribution to output depends only on the coordination activity of the entrepreneur. Of course, in the absence of choice variables any means of distributing the firm's product would be equally efficient. In this section, I introduce entrepreneurial effort, e, and assume that σ is a function of effort, rather than ability. The intuition is then straightforward: any payment scheme that assigns to workers a positive fraction of residual profits after payment of fixed wages reduces the entrepreneur's effort below the optimal level. This reduces revenues, and the firm is less valuable than to an entrepreneur who offers a purely fixed wage. Efficiency requires that the entrepreneur receives the full marginal product of effort, and an incidental consequence of this is that he also bears all the risk.

More formally, let $R_n(e)$ denote expected revenues for a firm with n workers, conditional on entrepreneurial effort. Assume R_n is an increasing concave function of effort,[13] and let the cost of effort be unity. Suppose the entrepreneur pays himself a fixed wage, v, and he offers workers a fixed wage, w, plus a fraction of residual profits, $\alpha\left(R_n(e)-v-wn\right)/n$, to each worker. The remainder of the residual profits go to the entrepreneur. If all agents are risk neutral, it must be the case that $wn=\left(n-\alpha\left(R_n(e)-v\right)\right)/(1-\alpha)$, so that the income expected from working for this firm equals the fixed wage offered by other firms. If the entrepreneur can commit to the level of effort when contracting with workers, the value of α does not matter. In this case, his expected net earnings are

$$\begin{aligned} w_e(e) &= (1-\alpha)\left(R_n(e)-v-\frac{n-\left(\alpha R_n(e)-v\right)}{1-\alpha}\right)+v-e \\ &= \left(R_n(e)-e\right)-n\,, \end{aligned} \tag{6}$$

which does not depend upon α. With commitment, any degree of profit sharing is equally efficient. This is not the case when the entrepreneur cannot commit to a particular level of effort. Let $w(\alpha)$ denote the wage set upon hiring workers. The entrepreneur's effort is the solution to

13. A simple parameterization for effort substitutes $e=s$ in (4). This yields an increasing quasi-concave function, which suffices for what follows.

$$e^*(\alpha) = \arg\max_e \left\{(1-\alpha)\left(R_n(e) - v - nw^*(\alpha)\right) + v - e\right\}, \tag{7}$$

yielding the first order condition $R_n'\left(e^*\right) = (1-\alpha)^{-1}$, so that $e^*(\alpha)$ and $R_n\left[e^*(\alpha)\right] - e^*(\alpha)$ are strictly decreasing in α. Given the fixed wage bill, $w\left[e^*(\alpha)\right]n = \left(n - \alpha R_n\left[e^*(\alpha)\right] - \alpha v\right)/(1-\alpha)$ the entrepreneur's net income is [from (6)] $w_e\left[e^*(\alpha)\right] = \left(R_n\left[e^*(\alpha)\right] - e^*(\alpha)\right) - n$, which is strictly decreasing in α.[14] Any reward scheme that does not fully reward the entrepreneur for his effort is therefore inefficient. Moreover, in contrast to Kihlstrom and Laffont (1979), entrepreneurs bear all risk even though everyone is risk neutral.

The analysis is more complicated if agents are risk averse, because there are three interacting effects of profit sharing. For a given level of effort, the entrepreneur's expected utility rises if he can substitute a fixed payment for part of his claim on residual profits. In order to do so, however, he must raise the expected earnings of workers to compensate them for accepting risk. Finally, of course, any change in the wage structure designed to diversify risk has direct consequences for the optimal amount of effort. Ignoring this last consequence (i.e. when effort is exogenous), there is likely to be profit sharing, except when entrepreneurs are sufficiently wealthy relative to employees. Because larger firms ceteris paribus yield larger residual profits for the entrepreneurs, this implies that the degree of profit sharing [the fraction of risk borne by the entrepreneur] declines [rises] with firm size. When effort is endogenous, the outcome depends on the productivity of entrepreneurial effort relative to the degree of risk aversion. If output is sufficiently sensitive to effort, then the entrepreneur will continue to bear all risk. In intermediate cases, a conditional form of Kihlstrom and Laffont's main result survives: if there is variation in risk aversion between agents with equal ability, then the least risk averse are the most appropriate entrepreneurs.

2.5 Risk and Return

Expected output per worker is independent of firm size, n, as is the entrepreneurial rent share. It then follows that entrepreneurial income is proportional to firm size. As the variance of output is proportional to firm size, the model generates a

14. Of course, n will also vary with α. But this serves simply to strengthen the finding. An entrepreneur with firm size n and profit sharing level $\alpha>0$ will prefer to reduce α to zero and increase his income. He may then decide to alter n and further increase income.

positive correlation between the mean and variance of entrepreneurs' earnings. This positive correlation is consistent with the conventional wisdom that risk aversion demands a risk premium, but in the present model it arises as an equilibrium response to variations in ability across risk-neutral entrepreneurs.

Using data from the PSID, Rosen and Willen (2002) confirm that the self-employed have higher and more volatile income than wage earners, but then proceed to question whether risk aversion can explain the apparent risk premium for self-employment. They find that the degree of relative risk aversion required to explain occupational choice exceeds conventional estimates by an order of magnitude,[15] and hence that risk attitudes cannot be a major determinant of the decision to become self-employed.[16] As previous empirical work imposes no *a priori* constraints on the distribution of abilities, the present model's link between ability and income variance may be a plausible explanation of Rosen and Willen's results.

2.6 General Equilibrium

The analysis so far has been of the "partial partial equilibrium" type: I have treated both the wage of workers and the price of output as exogenous data, and analyzed optimality in a single firm. In this subsection, I derive the equilibrium of an economy characterized by the production technology in (3). For compactness, I revert to the setting in which effort does not matter. All individuals may either produce one unit of work as a wage earner, or they may run a single firm with success that depends on ability, s.

Two equations determine equilibrium. The first is the zero profit condition for the marginal entrepreneur. Entrepreneurial earnings increase with s, so for any given price there is a critical skill level, $\overline{s}$, satisfying $E[w_e; p, \overline{s}] = 0$, such that

15. Among the less [more] educated, the coefficient of relative risk aversion required to explain employment is 129 [55], compared with conventional estimates of between 1 and 3. An alternative interpretation is that a model of occupational choice driven only by risk aversion explains only about one percent of the observed difference in consumption levels between the self-employed and wage earners.

16. A rather different puzzle is raised by Moskowitz and Vissing-Jørgensen (2002), who study the return to financial investment in public and private equity portfolios. They find that the returns are about the same but, because private equity portfolios tend to be much less diverse, their risk is much greater.

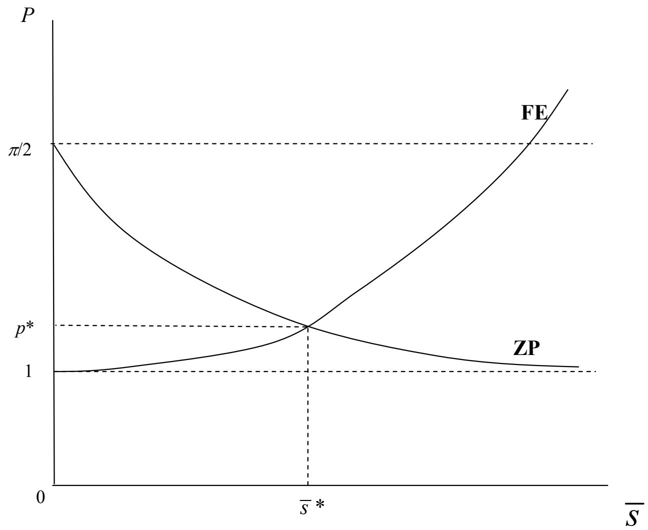

FIGURE 5. General Equilibrium.

individuals with $s \geq \overline{s}$ become entrepreneurs and the remainder become employees. As $E[w_e; p, s]$ rises with p, it is clear that $\overline{s}$ falls with p. Output per worker in the marginal firm tends to one as $\overline{s} \to \infty$ so, with a wage of unity, it follows that $\lim_{\overline{s}\to\infty} p(\overline{s}) = 1$. At the other extreme, as $\overline{s} \to 0$, output per worker tends to $2/\pi$ and for given n $\lim_{\overline{s}\to 0} E[w_e; p, \overline{s}] = n(2p/\pi - 1) - 1$. Thus, $\lim_{\overline{s}\to 0} p(\overline{s}) = \pi(n+1)/2n$. As n becomes unbounded for any $p \geq \pi/2$, $\lim_{\overline{s}\to 0} p(\overline{s}) = \lim_{n\to\infty} \pi(n+1)/2n = \pi/2$. The pairs of values $\{p, \overline{s}\}$ satisfying the zero-profit condition, $E[w_e; p, \overline{s}] = 0$, define a negatively-sloped line bounded between $p = 1$ and $p = \pi/2$. This is shown in Figure 5 as the line ZP.

The second equation is a full employment constraint. Let the population be N and let $F(s)$ denote the distribution of ability in the economy. Then a fraction $1 - F(\overline{s})$ of the population becomes entrepreneurs. As each entrepreneur with ability s employs $\varphi(p, \beta)s^{1/\beta}$ workers, the full employment constraint is

$$1 - F(\overline{s}) + \frac{1}{N}\int_{\overline{s}}^{\infty} \varphi(p, \beta)s^{1/\beta} dF(s) = 1\,. \tag{8}$$

Differentiating yields

$$\frac{\partial p}{\partial \overline{s}} = \frac{f(\overline{s})\left(N + \varphi(p,\beta)s^{1/\beta}\right)}{\varphi_p \int_{\overline{s}}^{\infty} s^{1/\beta} dF(s)} > 0\,, \tag{9}$$

where $\varphi_p = \partial\varphi / \partial p$. From (8), it is clear that p must increase without bound as $\overline{s} \to \infty$, else the integral term vanishes. At the other extreme, as $\overline{s} \to 0$, we require that $\varphi(p,\beta) \to 0$ for all s, which in turn requires that price falls to unity. The full employment constraint, indicated by the line FE in Figure 5, consequently traces a positively-sloped relationship between p and $\overline{s}$, beginning below ZP and then rising above it. As a result, there is a unique equilibrium.

There are few free parameters to concern ourselves about much in the way of comparative statics. One exception concerns changes in the distribution of skills. An increase in the ability of all agents induces every incumbent entrepreneur to increase employment. This violates the full employment constraint, and the change in ability must be offset at any given price by a reduction in the number of entrepreneurs. Thus, FE shifts right. The net result is a reduction in the equilibrium price, and an increase in $\overline{s}$. An improvement in the technology for organizing workers, captured parametrically by, for example, a reduction in β, shifts both lines. For any given value of s, profits rise as β falls, so the zero profit condition implies price must fall at each $\overline{s}$. Similarly, for each s, optimal employment rises, so price must also fall at each $\overline{s}$ in the full employment constraint. Thus, both FE and ZP shift down, inducing a reduction in price, but an ambiguous effect on the identity of the marginal entrepreneur.

3. Entrepreneurial Learning and Demand Growth

This section considers in partial equilibrium the implications of the model for firm growth and industry dynamics. Two engines of growth are considered. In the first, growth is driven by entrepreneurial learning. In the second, firms grow in response to secular increases in demand. Subsection 3.1 briefly considers growth of a single firm in an otherwise static industry, while subsections 3.2 and 3.3 consider some consequences of the two engines of growth. Throughout this section, I set $\beta = 1$ in order to reduce notation.

3.1 Firm Growth

The expected output of a firm run by an entrepreneur with skill s is

$$E\left[y(s)\right] = \int_0^1 \frac{sy\sqrt{2}e^{-\left(\cos^{-1}(y)\right)^2/2\varphi(p)^2}}{\sqrt{\pi\left(1-y^2\right)}\,\mathrm{erf}\left(\pi/\left(2\sqrt{2}\varphi(p)\right)\right)}\,dy$$

$$= \phi(p)s\,, \tag{10}$$

where $\phi'(p) > 0$. Output is clearly increasing in both s and p. Expected labor productivity is

$$\frac{E\left[y(s)\right]}{n(s)} = \frac{\phi(p)}{\varphi(p)} = \int_0^1 \frac{y\sqrt{2}e^{-\left(\cos^{-1}(y)\right)^2/2\varphi(p)^2}}{\varphi(p)\sqrt{\pi\left(1-y^2\right)}\,\mathrm{erf}\left(\pi/\left(2\sqrt{2}\varphi(p)\right)\right)}\,dy\,, \tag{11}$$

which is decreasing in p and (consistent with Proposition 2) independent of s. It immediately follows that the two engines of growth have different consequences for firm characteristics. If the entrepreneur improves his organizing skills over time, the resulting increases in s raise output for given n and, further, induce the firm to increase employment. However, the increase in employment exactly offsets the effect of higher ability on individual workers. As a result, growth driven by entrepreneurial learning is characterized by rising output and employment, but constant labor productivity. When growth is driven by improvements in market conditions, reflected here by an increase in the price, output and employment rise, but in this case firm growth is accompanied by declining labor productivity.

3.2 Growth from Learning

Assume there is a continuum of potential entrepreneurs with measure one, and that there exists an outside option paying a unit wage. Denote demand by $D(p) = Ag(p),\ \ g'(p) \le 0,$ so that A is a convenient scaling factor. To derive industry supply, recall that the marginal entrepreneur's skill level satisfies the zero profit condition, $p\phi(p)\overline{s} - (\varphi(p)\overline{s} + 1) = 0.$ Using (10), the industry supply curve is given by

$$Y(p) = \phi(p)\int_{\overline{s}(p)}^{\infty} s dF(s)\,, \tag{12}$$

where $\overline{s}(p) = \left(p\phi(p) - \varphi(p)\right)^{-1}$ is decreasing in p. Market equilibrium requires that

$$Ag(p) - \phi(p) \int_{\overline{s}(p)}^{\infty} s dF(s) = 0 \,. \tag{13}$$

To consider the effect of learning, replace s with $s + \alpha h(s)$ for some $\alpha \geq 0$, and some non-negative function $h(s)$. This characterization of course implies that all agents with ability s learn at exactly the same rate. If $h(s) = 0$ for all $s \leq \overline{s}$, only incumbent entrepreneurs learn, while if $h(s) > 0$ for any $s \leq \overline{s}$, some employees also learn about coordinating inputs. Market equilibrium then requires that

$$Ag(p) - \phi(p) \int_{\overline{s}(p) - \alpha h(\overline{s}(p))}^{\infty} \left[s + \alpha h(s) \right] dF(s) = 0 \,. \tag{14}$$

Differentiating (14) with respect to α and evaluating at $\alpha = 0$ yields

$$p'(\alpha) = - \frac{\phi(p) \int_{\overline{s}(p)}^{\infty} h(s) dF(s)}{\phi'(p) \int_{\overline{s}(p)}^{\infty} s dF(s) - \phi(p) \overline{s} f(\overline{s}) \overline{s}'(p) - Ag'(p)} < 0 \,. \tag{15}$$

The effects of learning are straightforward. The rightward shift in the supply curve is given in the numerator, and the induced decline in price (and increase in industry output) depends on the slopes of the supply and demand functions. Recalling that $\phi(p)s$ is the expected output of a firm indexed by skill level s, the term $\phi(p)h(s)d\alpha$ is the increase in expected output for a type s firm that is realized when price is held constant. The numerator sums this increase in output over all entrepreneurs. The slope of the supply curve is given by the first two terms in the denominator. The first of these sums over all incumbents the change in expected firm output induced by the decline in market price. The second is the change in output induced by a change in the marginal entrepreneur.

Equation (15) cannot tell us whether learning induces exit of incumbent entrepreneurs, or the creation of employee startups. To see this, note that the skill level of the marginal entrepreneur satisfies

$$\overline{s}(p(\alpha)) + \alpha h\left(\overline{s}(p(\alpha))\right) = \left(p(\alpha)\phi(p(\alpha)) - \varphi(p(\alpha))\right)^{-1}, \tag{16}$$

from which we obtain

$$\left.\frac{d\overline{s}}{d\alpha}\right|_{\alpha=0} = -h(\overline{s}) - \left(\frac{p\phi'(p)+\phi(p)-\varphi'(p)}{\big(p\phi(p)-\varphi(p)\big)^2}\right)p'(\alpha)\,. \tag{17}$$

Learning by the marginal agent has a direct, negative, effect, $-h(\overline{s})$, on the value of $\overline{s}$, which induces employee startups. Opposing this is a positive price effect on $\overline{s}$, which encourages exit by incumbents and discourages employee startups. Either effect may dominate. For example, under perfectly elastic demand, learning can only induce entry. On the other hand, if learning is limited to incumbent entrepreneurs, then it can only induce exit.

We already know that, holding price constant, an increase in entrepreneurial ability has no impact on labor productivity, while a decline in price with ability held constant raises productivity. As a result, supply-side growth will be associated in equilibrium with rising productivity among incumbents. However, average productivity in the *industry* may not rise if learning is limited to employees who then create startups, because mass is then being added to the lower end of the skill distribution of entrepreneurs.

3.3 Growth from Expanding Demand

The effects of growth driven by increased demand is captured by a rightward shift of the demand curve. For example, the price effect of an increase in A is

$$p'(A) = \frac{g(p)}{\phi'(p)\int_{\overline{s}(p)}^{\infty} s dF(s) + \phi(p)\overline{s}f(\overline{s})\overline{s}'(p) - Ag'(p)} > 0\,, \tag{14}$$

The interpretation is entirely standard: growth through expanding demand raises price and quantity; it induces increased output by incumbents at the cost of declining labor productivity; and it induces new entrants.

Entrepreneurs may seek to avoid the declining productivity induced by demand growth by selling the firm to a new owner with greater ability, who is more able to exploit the profit opportunities offered by expanding demand. A complete analysis of business transfers is beyond the scope of this paper; it requires, for example, that we specify how entrepreneurs are born and retire, as in the two models of Holmes and Schmitz (1990, 1995). But without undertaking a complete dynamic analysis, it is possible to draw a comparison with some of results from Holmes and Schmitz' analyses of business transfers. In their first model, agents differ in their ability to develop new businesses, but all agents are equally gifted at managing existing businesses. The most capable agents therefore are quick to sell businesses so they can get back to the task of new business development. The model predicts that firms that are transferred are on average better quality than firms that are not, and the agents who sell business are more able than those who do not. In their second model, there is no distinction between business development and management, and all agents are *ex ante* identical. The profitability of a business depends both on its quality and the quality of the match between the business and the entrepreneur. The model predicts that poor matches are likely to result in quick transfers of ownership. However, better business are also more likely to be sold, because the value of a good match is greater the better the quality of the business.

The present model is also consistent with the prediction that better quality firms (i.e. those with the greatest increases demand) are more likely to be transferred. What about the quality of the entrepreneur? The present model predicts that lower quality entrepreneurs are more likely to sell business, because entrepreneurs transfer businesses that have become too large given their ability to organize labor. Although the present model depends upon permanently differing abilities, this result is more in line with Holmes and Schmitz' (1995) second model.[17] Intuitively, an expansion of firm size in the face of fixed entrepreneurial ability is analogous to a decline in match quality.

To establish these results, consider the following simple thought experiment. There exist a set of incumbent entrepreneurs with skill levels drawn from the distribution $F(s)$, each of whom produces a unique product that initially sells at a common price, p_0. Subsequently, each entrepreneur experiences an improvement in the quality of his product as perceived by consumers, which translates into a rise in the price for his product. Let $G(p)$ denote the distribution of random price draws, with $G(p_0) = 0$ and $G(\overline{p}) = 1$ for some $\overline{p} \leq \pi/2$. There is also a set of potential buyers of these firms, which consists of agents with abilities, s', also drawn from the distribution $F(s')$. Each incumbent is matched at random with

17. And it is more in line with evidence indicating that transferred businesses were underperforming. For example, Lichtenberg and Siegel (1987) find that transferred businesses had lower initial levels of productivity but higher subsequent growth. McGuckin and Ngyen (1995) also report that the productivity of acquired plants rises following a takeover. See Holmes and Schmitz (1990, 1995) for discussion of further evidence.

one member of the set of potential buyers. Let $H(s'-s)$, denote the distribution of the difference between the abilities of the (potential) buyers and sellers, noting that $H(0)=½$. The incumbent may sell to his match with a transfer cost k, and he will do so if there is a net surplus to be had from the transfer. Recalling that expected profits for an entrepreneur are $E[w_e \mid s,p]=\left(p\phi(p)-\varphi(p)\right)s+1$, the gain from transferring a business from an agent with ability s to one with ability s' is $\left(p\phi(p)-\varphi(p)\right)(s'-s)-k$, where $\left(p\phi(p)-\varphi(p)\right)$ is positive and increasing in p. Hence, a transfer therefore takes place only if $s'-s\geq k/\left(p\phi(p)-\varphi(p)\right)$. That is, the potential buyer must have strictly greater ability than the incumbent, but the difference is decreasing in the quality of the firm's product. As a result, p tends to be higher than average for transferred businesses, while s tends to be lower than average.[18]

More formally, let $P(s'-s)$ be the lowest price that satisfies $s'-s\geq k/\left(p\phi(p)-\varphi(p)\right)$, so that $p^*=\max[p_0,P(s'-s)]$ is the minimum price at which a transfer takes place. It then follows that

$$
\begin{aligned}
&E\left[p \mid \left(p\phi(p)-\varphi(p)\right)\geq k/(s'-s)\right] \\
&\qquad = 2\int_0^\infty \frac{1}{1-F(p^*)}\int_{p^*}^{\bar{p}} p\,dG(p)\,dH(s'-s) \\
&\qquad \geq 2\int_0^\infty \int_{p_0}^{\bar{p}} p\,dG(p)\,dH(s'-s) \\
&\qquad = 2\int_0^\infty dH(s'-s)\int_{p_0}^{\bar{p}} p\,dG(p) \\
&\qquad = \int_{p_0}^{\bar{p}} p\,dG(p)\,, \qquad (15)
\end{aligned}
$$

so businesses that are transferred are on average better quality. Conversely, we have

18. Note also that higher growth rates for p induce more businesses to be transferred.

$$
\begin{aligned}
&E\left[s \mid s \le s' - k \,/\left(p\phi(p) - \varphi(p)\right)\right] \\
&\qquad = \int_{\underline{p}}^{\bar{p}} \int_{0}^{\infty} \frac{1}{F(\min[s', S(s',p)])} \int_{\underline{s}}^{\min[s', S(s',p)]} s dF(s) dF(s') dG(p) \\
&\qquad \le \int_{\underline{p}}^{\bar{p}} dG(p) \int_{0}^{\infty} \frac{1}{F(\min[s', S(s',p)])} \int_{\underline{s}}^{s']} s dF(s) dF(s') \\
&\qquad \le \int_{0}^{\infty} \frac{1}{F(s')} \int_{\underline{s}}^{s'} s dF(s) dF(s') \\
&\qquad = E\left[s \mid s \le s'\right] \\
&\qquad < E\left[s\right], \qquad (16)
\end{aligned}
$$

so the ability of entrepreneurs who transfer their business is on average lower.

4. Conclusions

This paper has explored one possible answer to the question: what is it that entrepreneurs (and managers) do? Perhaps more precisely, it has explored what there is to learn from developing a tractable model of one particular activity—coordinating the efforts of workers. Coordination is of course not the only task for the entrepreneur. Hellman (2007), for example, has recently analyzed the essential task of collecting resources. Nonetheless, coordination does seem to be an important activity and, as in previous work, there is likely something to learn from focusing on a single activity to the exclusion of others.

There are undoubtedly also a variety of ways in which one might model this coordination activity. The approach taken in this paper is especially tractable, but it also provides some substance, or ‘microstructure’ to the activities only implicitly present in Lucas’ (1978) classic paper. Moreover, it does so in a way that relates the ability of the entrepreneur to coordinate workers to the implications drawn from very different models of entrepreneurial behavior. In particular, the model provides an endogenous explanation for why entrepreneurs are risk bearers in equilibrium, an activity that was simply assumed to be a task of entrepreneurs in Kihlstrom and Laffont (1979). It also replicates some key implications about business transfers that Holmes and Schmitz (1990) derived in framework that focused on the inventive activity of entrepreneurs.

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